

## Unmeasured Investment and the Puzzling US Boom in the 1990s<sup>†</sup>

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*For the 1990s, the basic neoclassical growth model predicts a depressed economy, when in fact the US economy boomed. We extend the base model by introducing intangible investment and non-neutral technology change with respect to producing intangible investment goods and find that the 1990s are not puzzling in light of this new theory. There is microeconomic and macroeconomic evidence motivating our extension, and the theory's predictions are in conformity with US national accounts and capital gains. We compare accounting measures with corresponding measures for our model economy and find that standard accounting measures greatly understate the 1990s boom. (JEL E22, E23, O33, O47)*

The basic neoclassical growth model accounts well for the postwar behavior of the US economy prior to the 1990s, provided that variations in population growth, depreciation rates, total factor productivity (TFP), and taxes are incorporated.<sup>1</sup> The behavior of the 1990s, however, is strikingly at variance with this model, particularly in comparison with the boom in hours, but also in comparison with the behavior of most aggregate series that business cycle theorists study. To put it succinctly, the model predicts a depressed 1990s economy, when in fact it boomed.

In this paper, we extend the base model by introducing intangible investment and non-neutral technology change with respect to producing intangible investment goods and find that, in light of this new theory, the 1990s are not puzzling. Intangible investment is excluded from gross domestic product (GDP) because it is difficult to measure. Examples include research and development (R&D), advertising, and investments in building organizations. Some intangible investment is financed by owners of capital and is *expensed* rather than capitalized. Some intangible investment is financed by workers who are paid less than their marginal value product and receive some equity in their firm, which we refer to as *sweat equity*. These

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<sup>†</sup>To comment on this article in the online discussion forum, or to view additional materials, visit the articles page at <http://www.aeaweb.org/articles.php?doi=10.1257/mac.2.4.88>.

<sup>1</sup>See, for example, Kaiji Chen, Ayşe İmrohoroğlu, and Selahattin İmrohoroğlu (2007) and our technical Appendix (McGrattan and Prescott 2009).

investments are made with the expectation of realizing future profits or capital gains when the business goes public or is sold.

There is macroeconomic and microeconomic evidence that suggests that both types of unmeasured investment were abnormally high in the 1990s. The National Science Foundation's (NSF) (2007) report of R&D investment shows that industry R&D relative to GDP grew by 23 percent between 1994 and 2000. Carol Corrado, Charles Hulten, and Daniel Sichel (2005, 2006) include other categories of expenditures and conclude that over the 1990s, total intangible investment was large and increasing as a share of business output. If we look at patterns of the US national incomes, we find evidence of low factor incomes despite increased economic activity, suggesting that investment in expensed capital and sweat equity was high. Corporate profits were falling as output was rising, and compensation per hour was falling when hours were booming. Meanwhile, business capital gains, which are not included with national income, grew rapidly and accrued to households reporting the largest increase in hours in the Federal Reserve's Survey of Consumer Finances (SCF).

The fact that *measured* factor incomes were low when output and hours were booming is consistent with a theory that differentiates economic income and measured income, which need not move together and indeed did not move together in the 1990s. To uncover what actually happened during the 1990s, we use our extended theory and US national income and product account (NIPA) data. Specifically, we incorporate intangible investment into an otherwise basic neoclassical growth model. There are two activities in the business sector: producing final goods and services and producing intangible investment goods. We assume that hours allocated to the two activities are measured accurately, while reported incomes are understated by the amount of intangible investments of firm shareholders and worker-owners of businesses. We use the extended model to determine the path for intangible investment and show why including this type of investment is critical for understanding the boom in the US economy in the 1990s.

We allow the rates of technological change to differ across production of final goods and services and production of intangible investment goods, thus allowing for a high-technology boom in the sector producing intangible investment. This modeling choice is motivated by microevidence that shows large sectoral shifts into certain high-tech activities. The US Bureau of Economic Analysis (BEA) R&D satellite accounts, for example, show a large increase in the share of private business R&D done by information-technology (IT) producing companies. The Department of Labor's Current Population Surveys (CPS) show a shift of labor into IT-related industries and occupations.

Assuming that households equate wages and rental rates across production activities, we have a way to identify the TFP paths in our model's two sectors and to estimate the magnitude of intangible investment. We estimate that net intangible investment in the business sector was about 3 percent of GDP prior to 1990, rose to over 8 percent of GDP in the 1990s, and then returned to the level of the early 1990s in 2001.

To generate a boom in hours, we could have modified the basic growth model by introducing large and variable shocks to preferences for leisure, which is a

common practice in business cycle research.<sup>2</sup> However, doing so would have violated two criteria that we require to successfully resolve the puzzling 1990s boom—or any other puzzle for that matter. The first is the *input justification criterion*. By this, we mean that we require our exogenous inputs to be consistent with micro and macro empirical evidence. A large rise in preferences for leisure, on the other hand, cannot be justified by any observations on tax rates, tax credits, or welfare benefits.

The second of our two criteria for successfully resolving the puzzling 1990s boom is the *prediction criterion*. At a minimum, to satisfy this criterion, a theory's predictions must not be counterfactual. A stronger requirement—one that is satisfied by our theory—is to make correct predictions for data that were not used to set parameters and exogenous inputs. Thus, we do not follow the widely used practice in the business cycle literature of including the same number of exogenous inputs as observed time series, which is done to ensure a perfect fit between data and theory.

Here, there are two sequences of TFP parameters that are free in the analysis and many time series that must be in conformity with the theory. We find that the equilibrium paths of our extended theory are in close conformity with time series of both NIPA products and incomes and, most importantly, with the increase in capital gains that occurred in the second half of the 1990s. This increase was large, with the average real gains going from 6 percent of GDP in the period 1953–1994 to 12 percent of GDP in the period 1995–2003. Data on factor incomes and capital gains are not used to identify the TFP parameters. In contrast, a theory based on a large shift in preferences for leisure during the 1990s does not account for the observed changes in factor incomes and capital gains.<sup>3</sup>

After demonstrating that the model's predictions are in conformity with US time series, we use the model to compare current accounting measures for investment and labor productivity with corresponding measures that include expensed and sweat investment. If expensed and sweat investments are included, the model predicts an earlier and larger boom in productivity than current accounting measures would indicate. Based on this prediction, we conclude that ignoring these two types of intangible investments distorts the true picture of the US economy in the 1990s.

Our findings show that standard productivity measures greatly understate the actual rise in labor productivity, whether we consider the overall economy or the business sector. Our analysis is not subject to the criticism of Erik Brynjolfsson and Loren M. Hitt (2000), who point out that intangible investment is “not well captured by traditional macroeconomic measurement approaches.” Here, we explicitly model the intangible investment. Accounting for intangible investment,

<sup>2</sup> See, for example, Robert E. Hall (1997); Yongsung Chang and Frank Schorfheide (2003); Jordi Galí (2005); Diego Comin and Mark Gertler (2006); Galí, Gertler, and J. David López-Salido (2007); James A. Kahn and Robert W. Rich (2007); Frank Smets and Rafael Wouters (2007); and Peter N. Ireland and Scott Schuh (2008), who point out that these shocks proxy for variations in tax rates and other labor market distortions, which are especially important in accounting for changes in hours. In McGrattan and Prescott (2009), we show that labor tax rates do account for much of the cyclical variation in hours prior to the 1990s but not during the 1990s.

<sup>3</sup> See McGrattan and Prescott (2009) for details.

we find that the boom in business productivity began earlier and was bigger than standard statistics show. Over the period 1993–2000, the difference in labor productivity growth due to the inclusion of intangible investment is 0.8 percent per year.

Our paper is complementary to the work in the growth-accounting literature that shows information technology was an important engine of growth during the 1990s.<sup>4</sup> This work suggests that investment-specific technological change could be an important contributor to the 1990s boom. However, if we abstract from intangible capital and permit technological change to be non-neutral between tangible investment and consumption, we do not resolve the puzzling 1990s boom.<sup>5</sup> We show that the fact that part of output is not counted is critical to understanding the boom in hours and the large rise in capital gains.

Urban J. Jermann and Vincenzo Quadrini (2007) propose an alternative theory of the 1990s boom in which expectations of higher future productivity loosen financial constraints on small firms and significantly boost their current productivity. Because they assume that there is no mismeasurement of outputs, Jermann and Quadrini's (2007) theory predicts a simultaneous boom in hours, output, wages, and labor productivity. A puzzling aspect of the 1990s boom, however, was the fact that these series did not rise simultaneously; hours rose while compensation per hour fell and before the rise in measured output and labor productivity.

The paper is organized as follows. In Section I, we establish that the basic theory, which abstracts from unmeasured investment, generates strongly counterfactual predictions. In particular, the basic growth model does not generate a boom in the 1990s. Two-sector variations of the basic growth model—allowing for non-neutral technical change in business versus nonbusiness, or consumption versus tangible investment—show some improvement in the model's predictions but do not successfully resolve the puzzle. In Section II, we summarize the evidence of increased intangible investment which motivates our extension of the basic theory. The extended theory includes expensed and sweat investment, and in Section III, we assess its predictions. In Section IV, we reevaluate the performance of the US economy in the 1990s through the lens of the extended theory. Conclusions are found in Section V.

## I. Predictions of the Basic Theory without Intangible Investment

Our starting point is the basic growth model used in the study of business cycles. This model abstracts from intangible investment. We treat TFP, tax rates, and population exogenously. We then use US data for the 1990s to establish that there are large deviations from the basic model, indicating that this model must be abstracting from something important. We show that the nature of the deviations points to unmeasured investments.

<sup>4</sup> See, for example, Dale W. Jorgensen, Mun S. Ho, and Kevin J. Stiroh (2008), and Stephen D. Oliner, Sichel, and Stiroh (2007).

<sup>5</sup> See, for example, Andreas Hornstein and Per Krusell (1996), and Jeremy Greenwood, Zvi Hercowitz, and Krusell (1997).

### A. The Basic Growth Model

In the standard one-sector growth model, given the initial capital stock  $k_0$ , the problem for the stand-in household is to choose consumption  $c$ , investment  $x$ , and hours  $h$  to maximize

$$E \sum_{t=0}^{\infty} \beta^t U(c_t, h_t) N_t,$$

subject to the constraints

$$\begin{aligned} c_t + x_t &= r_t k_t + w_t h_t - \tau_{ct} c_t - \tau_{ht} w_t h_t - \tau_{kt} k_t - \tau_{pt}(r_t k_t - \delta k_t - \tau_{kt} k_t) \\ &\quad - \tau_{dt} \{r_t k_t - x_t - \tau_{kt} k_t - \tau_{pt}(r_t k_t - \delta k_t - \tau_{kt} k_t)\} \\ k_{t+1} &= [(1 - \delta)k_t + x_t]/(1 + \eta), \end{aligned}$$

where variables are written in per capita terms and  $N_t = N_0(1 + \eta)^t$  is the population in  $t$ . Capital is paid rent  $r_t$ , and labor is paid wage  $w_t$ . Households discount future utility at rate  $\beta$ , and capital depreciates at rate  $\delta$ . Taxes are levied on consumption at rate  $\tau_c$ , labor income at rate  $\tau_h$ , tangible capital (that is, property) at rate  $\tau_k$ , profits at rate  $\tau_p$ , and capital distributions at rate  $\tau_d$ . Note that taxable income for the tax on profits is net of depreciation and property tax, and taxable income for the tax on distributions is net of property tax and profits tax.

The aggregate production function is

$$Y_t = A_t F(K_t, H_t),$$

where capital letters denote aggregates. The parameter  $A_t$  is TFP that varies over time. The firm rents capital and labor. If profits are maximized, then the rental rates are equal to the marginal products. The goods market clears so  $N_t(c_t + x_t) = Y_t$ . Here,  $c$  includes both private and public consumption, and  $x$  includes both private and public investment.

We now show that this basic growth model generates grossly counterfactual predictions during the 1990s. To do so, we compute the model's equilibrium path with households having perfect foresight of future changes in tax rates, TFP, and populations. In Appendix A, we discuss our US data sources and the adjustments we make to construct the empirical counterparts of the model variables. In Appendix B, we describe and motivate the parameterization we use for the model. The parameter values used to compute the equilibrium path here are summarized in Table B1 under "One-sector model, no intangible investment." The paths for TFP and tax rates are reported in Table B2.<sup>6</sup>

Our estimate for TFP is US GDP divided by  $F(K, H)$ , with  $K$  equal to the stock of tangible capital,  $H$  equal to total hours, and  $F(K, H) = K^\theta H^{1-\theta}$ . The stock of

<sup>6</sup>In McGrattan and Prescott (2009), we demonstrate that the perfect foresight assumption is innocuous by comparing the results to stochastic simulations. We also demonstrate the robustness of the results by varying parameters.

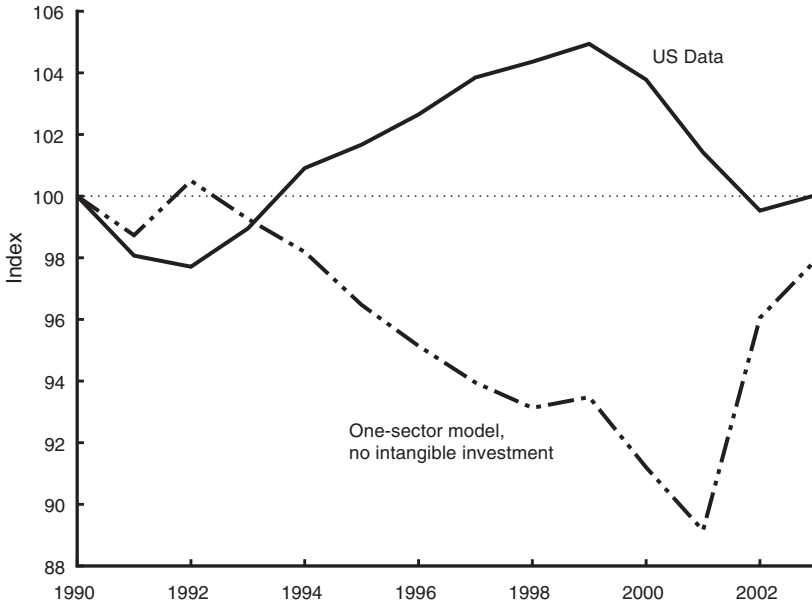


FIGURE 1. US AND BASIC MODEL PER CAPITA HOURS WORKED  
(Annual, 1990 = 100, 1990–2003)

capital is derived by applying the perpetual inventory method to data on tangible investment. The tax rate changes we consider are variations in the labor tax rates  $\tau_{ht}$  and consumption tax rates  $\tau_{ct}$ , as constructed in Prescott (2004) with data from US national accounts. During the 1990s, there was little change in legislation affecting capital taxation, and therefore we simply fix the rates  $\tau_{kt}$ ,  $\tau_{pt}$ , and  $\tau_{dt}$ .

The utility flow function is

$$U(c, h) = \log(c) + \psi \log(1 - h),$$

which is standard in the business cycle literature. We choose the level of capital tax rates, the depreciation rate  $\delta$ , and the utility parameter  $\psi$  so that the model's consumption share, investment share, factor inputs, and tax revenues are consistent with US levels in 1990. (See Appendices A and B for details.)

In Figure 1, we plot the model's predicted per capita hours of work along with the US actual per capita hours, indexed so that 1990 equals 100. The difference between the series is striking. Actual per capita hours rose 8 percent between 1992 and 1999, whereas the predicted series falls significantly during the same period.

In Figure 2, we plot the model's predicted output along with US real GDP. Both series are adjusted for population and a secular trend of  $1.02^t$ . Although the boom in output was not quite as large as the boom in hours, the model predicts that the economy should have been depressed. This counterfactual prediction arises from the fact that the tax rates on labor rose during the 1990s and economy-wide TFP was below trend during most of the decade.

The basic model has neutral TFP change with respect to the business and non-business sectors. In fact, TFP change was non-neutral for these sectors. A question

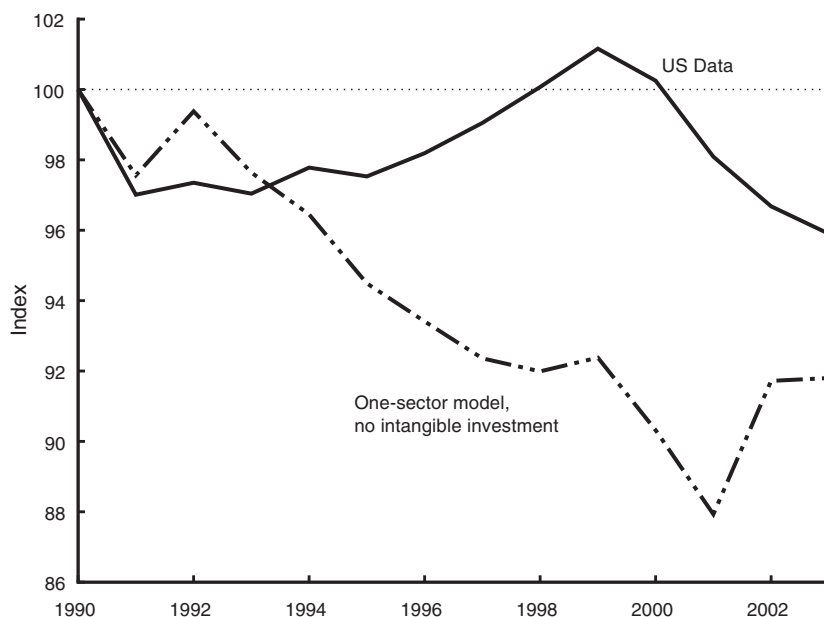


FIGURE 2. US AND BASIC MODEL PER CAPITA REAL GDP  
(Annual, series divided by 1.02', 1990 = 100, 1990–2003)

that arises is whether modeling this non-neutrality of TFP change could significantly narrow the large deviation from theory. In the two-sector version of the model, households solve the same problem, although they now have to allocate capital and labor to two technologies, “business” and “nonbusiness.” (See McGrattan and Prescott 2009 for model details.)

In Appendix A, we describe how we categorize business and nonbusiness activity. In Appendix B, we describe and motivate the parameterization for this version of the model. Parameter values used to compute the equilibrium path here are summarized in Table B1 under “Two-sector model, no intangible investment.” As before, the paths for TFP and tax rates are exogenous. These are reported in Table B2. We assume that households in the model make exactly the same choices for nonbusiness activity as US households. We simply set the time paths of nonbusiness hours, investment, and value added exogenously and use US time series. We treat this sector exogenously because it is quite small compared to the business sector. Furthermore, if there are any deviations between the model’s predictions and US data, we can attribute these deviations to our model of the business sector, which did have a boom in TFP.

We find that modeling the non-neutrality of TFP in business and nonbusiness activity does not resolve the puzzling 1990s boom. Model predictions are still far from observations. In particular, output is still predicted to be below trend throughout the decade, and per capita hours are still predicted to be low. (Figures of these series and others are in McGrattan and Prescott 2009.) The boom in business-sector TFP is too small and too late. Clearly, something else gave rise to the puzzling behavior of the US economy in the 1990s.

Another technological non-neutrality that could potentially resolve the puzzling 1990s boom is that between tangible investment and consumption. In McGrattan and Prescott (2009), we extend the basic one-sector model to allow for investment-specific technology change and use NIPA price deflators to construct a relative price between tangible investment and consumption. The model with investment-specific technology change predicts a counterfactually large rise in hours between 1991 and 1992 and a decline between 1992 and 2001. The overall pattern is similar to the prediction in Figure 1, although the depression in hours is less severe. Including the non-neutrality does improve the predictions for (chain-weighted) real GDP, but large deviations between theory and data still exist. (See McGrattan and Prescott 2009 for further details.)

### B. Investigating the Deviation

Why are the model's predictions so far off for the basic neoclassical growth model? The main reason is the behavior of TFP and tax rates. Given the behavior of these inputs, the model predicts an after-tax real wage,  $(1 - \tau_h)w$ , below its secular trend. Not surprisingly, then, the model predicts that hours are low and output is below trend. Adding non-neutral technical change in the business sector or in tangible investment does not change this prediction.

We turn next to evidence that using the wrong measure of output and understating labor productivity (and, therefore, the after-tax real wage) accounts for the large deviation between theory and data. The mismeasurement stems from abnormally large unmeasured intangible investment during this period. Standard measures of output growth are distorted when the importance of intangible investments grows.

## II. Evidence of Increased Intangible Investment

We present evidence that suggests that unmeasured intangible investment was abnormally high during the 1990s and that technological change was non-neutral with respect to the production of intangible investment. One type of evidence is related to the behavior of factor incomes and capital gains, the other type to the technology boom going on during the period.

### A. Low Compensation and Corporate Profits

If all incomes were included in the national accounts, we would expect both compensation per hour and profits to be high during a boom. Because intangible investments are expensed in the NIPA, measurements of factor incomes are understated to a greater extent in periods when these investments are high. An examination of the US national accounts reveals that, in fact, compensation per hour and profits were low during the boom period, suggesting that both unmeasured expensed and sweat investment was abnormally high. An examination of capital gains in the Federal Reserve's Flow of Funds reveals that capital gains in the 1990s were also abnormally high.



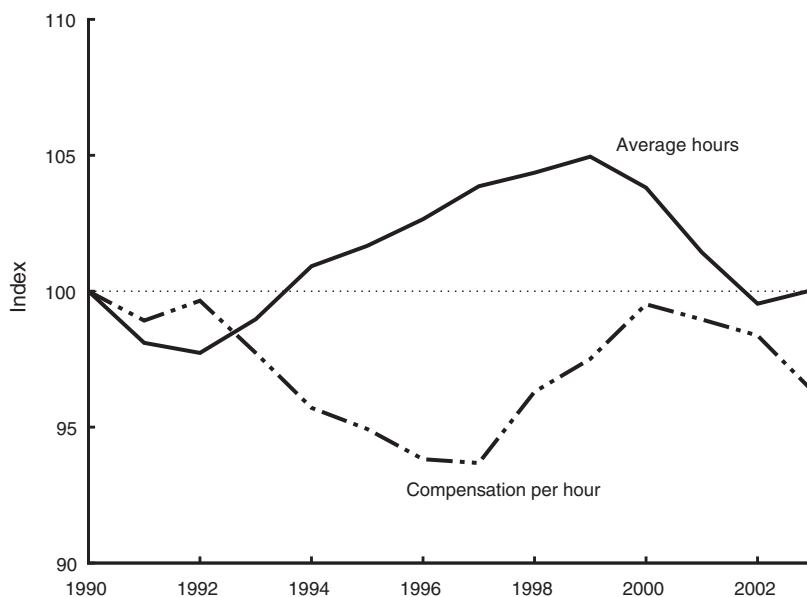


FIGURE 3. AVERAGE WEEKLY HOURS WORKED BY NONINSTITUTIONAL POPULATION, AGE 16–64, AND NIPA REAL COMPENSATION PER HOUR (Annual, compensation divided by  $1.02^t$ , 1990 = 100, 1990–2003)

In Figure 3, we plot average weekly hours of work for the noninstitutional population, age 16–64, which is the same series plotted in Figures 1 and 2. We also plot the wage rate corresponding to these hours, which is computed as follows. We take NIPA compensation and deflate it by the GDP deflator. We then correct for population growth by dividing real compensation by this population. Finally, because there is technological growth, we divide the wage rate by the factor  $1.02^t$ , where  $t$  indexes time.<sup>7</sup> For all of the 1990s, NIPA real compensation per hour detrended in this way is below the 1990 level, despite the boom in hours.<sup>8</sup> On the other hand, capital gains reported in the Federal Reserve’s Flow of Funds rose significantly during the 1990s, with real gains averaging 6 percent of GDP between 1953 and 1994 and 12 percent of GDP between 1995 and 2003 (see McGrattan and Prescott 2009). These observations are consistent with abnormally high sweat investment.

In Figure 4, we compare NIPA GDP and corporate profits, both deflated by the GDP deflator and adjusted for population and a secular trend of  $1.02^t$ . We see that profits fall in the late 1990s when GDP is high relative to trend. With the rise in GDP came an even faster rise in R&D. The National Science Foundation (2007, Tables 1 and 13) reports that R&D performed by industry increased 70 percent between 1994 and 2000, whereas GDP rose by 39 percent. These observations are consistent with abnormally high investment that is expensed from corporate profits.

<sup>7</sup> The particular choice of 1.02 for the secular trend does not affect any results, but makes it easier to see the patterns in this and later figures.

<sup>8</sup> In earlier work (McGrattan and Prescott 2005a), we abstract from sweat equity investment and treat NIPA compensation as true labor income. Doing so reduces the estimate of intangible investment.

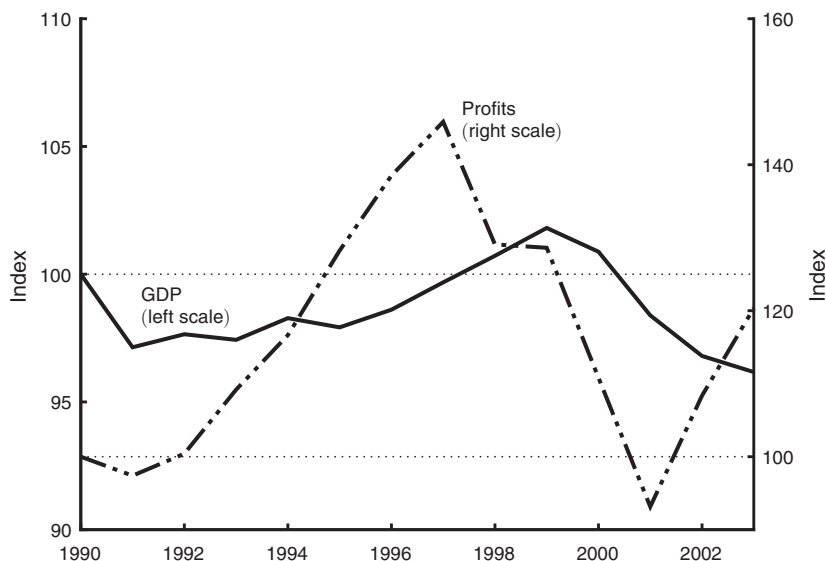


FIGURE 4. NIPA REAL PER CAPITA GDP AND CORPORATE PROFITS  
(Annual, series divided by 1.02<sup>t</sup>, 1990–2003)

### B. The Technology Boom

The 1990s was a period of rapid growth in R&D investment, hours of work, capital gains, and technological advances. In this section, we show this was especially true for industries and occupations related to information technology.

In Figure 5, we plot data from the BEA R&D satellite accounts that show the share of private business investment in R&D for eight four-digit IT industries covering computer and electronic product manufacturing, software publishers, and computer systems design and related services.<sup>9</sup> Between 1992 and 2002, this IT share of R&D investment doubled.

We next show that employment and hours in IT-related activities rose disproportionately during the 1990s. In Figure 6, we plot the employment in industries that the Department of Commerce's Economics and Statistics Administration (ESA) categorizes as IT-producing as a share of all employment in private industries. The ESA estimates are derived from the Bureau of Labor Statistics (BLS) Current Employment Statistics survey data.<sup>10</sup> In 1993, IT-producing industries accounted for only 3.9 percent of private employment. Between 1993 and 2000, employment in IT-producing industries rose 52 percent, while employment in private industries rose only 21 percent.

<sup>9</sup> The 2002 NAICS codes for these industries are 3341, 3342, 3344, 3345, 3343, 3346, 5112, and 5415. The source of the BEA R&D expenditures is the National Science Foundation.

<sup>10</sup> See Appendix table 2.1 of *Digital Economy* (2003), which contains the list of industries categorized as IT-producing.

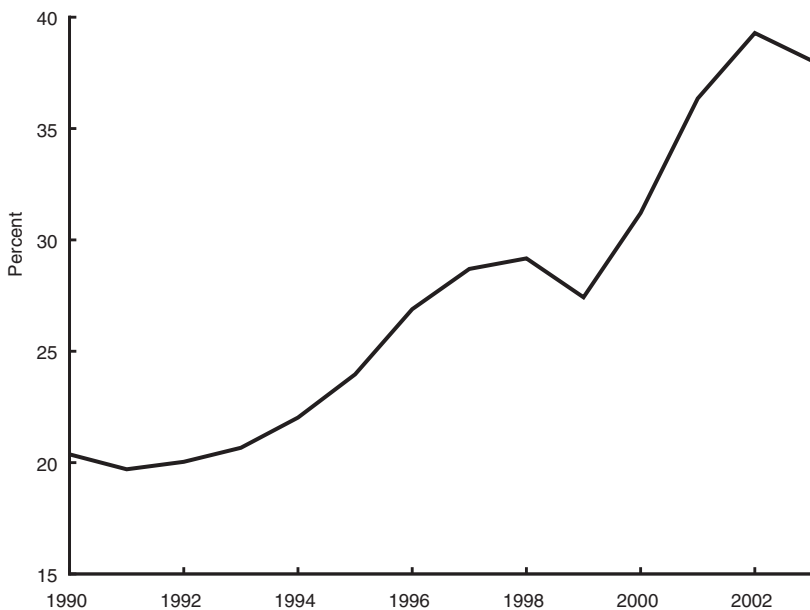


FIGURE 5. SHARE OF PRIVATE R&D BY IT PRODUCERS  
(Annual, 1990–2003)



FIGURE 6. SHARE OF PRIVATE EMPLOYMENT BY IT PRODUCERS  
(Annual, 1993–2002)

From the March supplements of the CPS, we can estimate the hours contribution of IT-related industries and occupations. We also condition on education levels in order to categorize IT-related work as high skilled or low skilled. Specifically, we

split workers into two groups: the educated in IT-related work and the remainder.<sup>11</sup> If the first group of workers includes all those with at least one year of college doing work in IT-producing or IT-related occupations, then we find that this group accounted for 42 percent of the increase in hours between 1992 and 2000, even though they accounted for only 6.6 percent of the population in 1992. If we expand the set of industries to include those categorized by Oliner, Sichel, and Stiroh (2007) as intensively using IT capital services, then we find that the group accounts for 57 percent of the increase in hours between 1992 and 2000, even though they accounted for only 12.6 percent of the population in 1992.<sup>12</sup>

Despite strong growth in IT employment and hours during the 1990s, median weekly earnings in core IT occupations were not above average. In *Digital Economy* (2000), the ESA reports that the average growth rate in the employment of computer scientists, computer engineers, and systems analysts was 15 percent per year between 1992 and 1998, much faster than the average rate of 2.5 percent for all occupations. The average growth rates in median earnings, however, were the same for these IT workers and all workers: 2.7 percent per year between 1992 and 1998. One possible explanation put forth by the ESA is that businesses use nonwage benefits such as stock options to attract employees. Stock options show up as compensation later when options are exercised. Another explanation is that these workers owned shares in the business and earned capital gains.

We can use the Federal Reserve's Survey of Consumer Finances (SCF) to link increased hours and business capital gains. Although the SCF public datasets do not report the same occupational and industry detail as the CPS, we do know from the CPS survey that many of the workers in IT-related occupations fall in the SCF occupational category "managerial and professional."<sup>13</sup> For capital gains, the data are provided at the household level. Thus, we divide the household gains between the respondent and the spouse (if there is one) in proportion to the number of hours that they work. When we split the sample of individuals into those with one year of college that work in managerial and professional occupations and the remainder, we find that the first group—which is only 17 percent of the population in the 1992 survey—accounts for 78 percent of the change in total hours between 1992 and 2001 and 68 percent of the change in business capital gains.

The fact that the booms in R&D investment and hours were concentrated in certain high-tech activities suggests that the main driving force in this period was technological in nature rather than a change in labor market distortions or preferences. There is also much anecdotal evidence suggesting rapid advances in technologies. For example, Mark Doms (2004) reports that Intel processor speeds increased 4.6 percent per month in the period 1997–2000, and fiber-optic throughput rose from 2.5 gigabits to 400 gigabits between 1995 and 2000. These advances sparked the interest of

<sup>11</sup> See Steven Ruggles et al. (2004) for details on the microdata series. In McGrattan and Prescott (2009), we provide details about the variables and filters that we use. We also do sensitivity analysis, changing our definition of "IT-related."

<sup>12</sup> See McGrattan and Prescott (2009) for CPS industry codes categorized as "IT-producing" and "IT-using" and the list of CPS occupation codes categorized as "IT-related."

<sup>13</sup> In McGrattan and Prescott (2009), we also report similar results for an expanded set of occupations that includes all IT-related occupations. Unfortunately, the SCF aggregation prevents us from separating out IT-related industries.

productivity analysts surveyed by Jorgensen, Ho, and Stiroh (2008), who find that the speedup in the second half of the 1990s was driven by information technology.

After 2000, the United States experienced large declines in R&D investment, hours of work, and capital gains. According to Elise A. Couper, John P. Hejkal, and Alexander L. Wolman (2003) and Doms (2004), the telecommunications sector contributed significantly to this *technology bust*. Part of the problem was that the demand for long-haul fiber capacity was not as great as anticipated. Another problem was what Federal Communications Commission (FCC) chairman Michael Powell called “the problem of legal instability in the court system” (Powell 2002). The Telecommunications Act of 1996 brought competition to local telephone service, but it generated many legal battles as well. According to Couper, Hejkal, and Wolman (2003), the regulatory uncertainty has discouraged telecommunications companies from undertaking the large-scale investments needed for fiber-optic service to individual residences.

In summary, the microeconomic and macroeconomic evidence indicates that intangible investment was abnormally high during the 1990s and that the boom was concentrated in particular activities. This motivates an extension of the basic model with both tangible and intangible investment and non-neutral technological change that gives rise to an increase in the importance of intangible investment.

### III. Predictions of the Extended Theory with Intangible Investment

We now extend the basic growth model to include intangible investment and non-neutral technological change with respect to production of final goods and services and production of new intangible investment goods. Intangible investments are made by businesses, so the extended model distinguishes business and nonbusiness activity. We start by describing the technologies available to businesses, the optimal business size, and the aggregate production technology. The household problem remains the same as in Section II except for an additional investment choice. We examine the extended model’s predictions and show that these predictions are in conformity with US observations during the 1990s.

#### A. Extensions

The aggregate production technology is characterized by the two aggregate production relations:

$$(1) \quad y_{bt} = A_t^1 (k_{Tt}^1)^\theta (k_{It})^\phi (h_t^1)^{1-\theta-\phi}$$

$$(2) \quad x_{It} = A_t^2 (k_{Tt}^2)^\theta (k_{It})^\phi (h_t^2)^{1-\theta-\phi}.$$

Firms produce business output  $y_b$  using their intangible capital  $k_I$ , tangible capital  $k_T^1$ , and labor  $h^1$ . Firms produce new intangible capital  $x_I$ —such as new brands, new products R&D, patents, etc.—using intangible capital  $k_I$ , tangible capital  $k_T^2$ , and labor  $h^2$ .

Note that  $k_I$  is an input to both business sectors; it is not split between them as is the case for tangible capital and labor. A brand name is used both to sell final goods and services and to develop new brands. Patents are used by the producers and the researchers. The aggregation theory underlying this technology is developed in Appendix C.<sup>14</sup>

Given  $(k_{T0}, k_{I0})$ , the stand-in household maximizes

$$(3) \quad E \sum_{t=0}^{\infty} \beta^t [\log c_t + \psi \log(1 - h_t)] N_t$$

subject to

$$(4) \quad \begin{aligned} c_t + x_{Tt} + q_t x_{It} &= r_{Tt} k_{Tt} + r_{It} k_{It} + w_t h_t + \zeta_t \\ &- \tau_{c_t} c_t - \tau_{h_t} (w_t h_t - (1 - \chi) q_t x_{It}) - \tau_{k_t} k_{Tt} \\ &- \tau_{p_t} \{r_{Tt} k_{Tt} + r_{It} k_{It} - \delta_T k_{Tt} - \chi q_t x_{It} - \tau_{k_t} k_{Tt}\} \\ &- \tau_{d_t} \{r_{Tt} k_{Tt} + r_{It} k_{It} - x_{Tt} - \chi q_t x_{It} - \tau_{k_t} k_{Tt} \\ &- \tau_{p_t} (r_{Tt} k_{Tt} + r_{It} k_{It} - \delta_T k_{Tt} - \chi q_t x_{It} - \tau_{k_t} k_{Tt})\} \end{aligned}$$

$$(5) \quad k_{T,t+1} = [(1 - \delta_T) k_{Tt} + x_{Tt}] / (1 + \eta)$$

$$(6) \quad k_{I,t+1} = [(1 - \delta_I) k_{It} + x_{It}] / (1 + \eta).$$

As before, all variables are in per capita units and there is growth in population at rate  $\eta$ . Consumption  $c$  includes both private and public consumption, and tangible investment  $x_T$  includes both private and public investment. The relative price of intangible investment and consumption is  $q$ . The rental rates for business tangible and intangible capital are denoted by  $r_T$  and  $r_I$ , respectively, and the wage rate for labor is denoted by  $w$ . Inputs are paid their marginal products. The tax system is the same as in the standard model. Other income is denoted by  $\zeta$  and is exogenous in the household's decision problem. Other income includes government transfers and nonbusiness capital income net of taxes and investment. Nonbusiness labor income is included in  $wh$ .

As before, we treat hours, investment, and output in the nonbusiness sector exogenously because this sector is not important for the issues being addressed. To be precise, we set hours  $\bar{h}_n$ , investment  $\bar{x}_n$ , and output  $\bar{y}_n$  in the model's nonbusiness sector in each period equal to US quantities. Measured output, which corresponds to GDP, is the sum of  $y_b$  and  $\bar{y}_n$ . Measured tangible investment is the sum of business tangible investment  $x_T$  and nonbusiness tangible investment  $\bar{x}_n$ . Measured hours  $h$  is the sum of business hours  $h_b = h^1 + h^2$  and nonbusiness hours  $\bar{h}_n$ .

<sup>14</sup> In McGrattan and Prescott (2009), we explore specifications in which the income shares are not equal in the two activities.

Let  $\chi$  denote the fraction of intangible investment financed by capital owners. The amount  $\chi qx_I$  is *expensed investment*, financed by the capital owners who have lower accounting profits the greater this type of investment. The amount  $(1 - \chi)qx_I$  is *sweat investment*, financed by workers who have lower compensation the greater this type of investment.<sup>15</sup>

GDP in the economy is the sum of total consumption (public plus private) and tangible investment (public plus private) for business and nonbusiness; in per capita terms, GDP is  $c + x_T + \bar{x}_n$ . Gross domestic income (GDI) is the sum of all labor income less sweat investment  $wh - (1 - \chi)qx_I$ , business capital income less expensed investment,  $r_T k_T + r_I k_I - \chi qx_I$ , and nonbusiness capital income (which is found residually as the difference between GDP and the other components of GDI). Thus, a NIPA accountant in this economy would measure the following product and income:

$$\begin{aligned} \text{NIPA product} &= c + x_T + \bar{x}_n \\ \text{Consumption} &= c \\ \text{Investment} &= x_T + \bar{x}_n \\ \text{NIPA income} &= y_b + \bar{y}_n \\ \text{Business profits} &= (r_T - \tau_k - \delta_T)k_T + r_I k_I - \chi qx_I \\ \text{Business wages} &= wh_b - (1 - \chi)qx_I \\ \text{Business depreciation} &= \delta_T k_T \\ \text{Business production tax} &= \tau_k k_T \\ \text{Nonbusiness income} &= \bar{y}_n. \end{aligned}$$

### B. A Resolution of Seemingly Low Wages

We showed earlier that there is a large deviation between predictions of the basic one-sector and two-sector growth models and US data. The models predict that after-tax real wages in the 1990s should have been below trend, leading to low per capita hours and output below its secular trend. With our extended model, the measure of the real wage is different and is consistent with the behavior of output and hours.

The standard model measure of the business real wage is

$$(7) \quad \hat{w}_t = (1 - \theta)y_{bt}/(h_t^1 + h_t^2),$$

where  $\theta$  is the capital share,  $y_{bt}$  is measured business value added, and  $h_t^1 + h_t^2$  is total business hours. The problem with the measure of labor productivity on the right side of equation (7) is that some hours are used to accumulate intangible capital. The hours used to produce  $y_{bt}$  are  $h_t^1$  and, therefore, the real wage measure is

$$(8) \quad w_t = (1 - \theta - \phi)y_{bt}/h_t^1,$$

<sup>15</sup> In the absence of informational or financial constraints, the choice depends in a knife-edge way on the tax treatment of the expensed and sweat investment. This is analogous to the result of Merton H. Miller (1977) for the debt-equity choice.

where  $y_{bt}/h_t^1$  is labor productivity in production of final goods and services. The labor input  $h_t^2$  is used to produce output  $q_t x_{it}$  and is not part of the labor input in producing  $y_{bt}$ . If the relative size of  $h_t^2$  relative to  $h_t$  increases, then  $w_t/\hat{w}_t$  increases and the percentage understatement of true wages becomes more severe.

The evidence presented earlier suggests that advances in technology were particularly large in activities related to intangible production. This would imply an increase in  $A_t^2/A_t^1$ . Our hypothesis is that  $A_t^2/A_t^1$  did indeed increase significantly and that such an increase leads to an increase in the relative hours allocated to producing intangible investments, namely  $h_t^2/h_t$ .

### C. Identifying Total Factor Productivities

In order to identify total factor productivities, the magnitude of the inputs and the outputs to the production functions must be determined. This requires determining the split of hours and tangible capital between two production activities in the business sector and the magnitude of intangible investment and capital.

To determine how much labor is allocated to the two production activities, we use the fact that the after-tax real wage rate is equal to the marginal rate of substitution between leisure and consumption,  $\psi(1 + \tau_{ct})c_t/(1 - h_t)$ . We have observations on consumption  $c$ , total hours  $h$ , business value added  $y_b$ , and tax rates. We use these observations to determine hours in production of final goods and services as follows:

$$(9) \quad h_t^1 = \left( \frac{1 - \theta - \phi}{\psi} \right) \left( \frac{1 - \tau_{ht}}{1 + \tau_{ct}} \right) \left( \frac{y_{bt}}{c_t} \right) (1 - h_t).$$

Hours in the accumulation of intangible capital is determined residually,  $h_t^2 = h_t - h_t^1 - \bar{h}_{nt}$ . Equation (9) is simply a rewriting of the household's intra-temporal condition relating the marginal rate of substitution between leisure and consumption to  $(1 - \tau_h)w$  using  $w$  in (8).

Equating the marginal products of labor in the two activities yields an equation that can be solved for  $q_t x_{it}$  as a function of the hours  $h^1$  and  $h^2$  determined above and business value added  $y_b$  which is observed,

$$q_t x_{it} = \frac{y_{bt}}{h_t^1} h_t^2.$$

Notice that our derivation of intangible expenditures relies heavily on theory and observations on consumption, total hours, business value added, and tax rates. This method of indirect inference has an advantage over direct measurement when some or all of the investment is not or cannot be measured.<sup>16</sup>

The allocation of tangible capital across the two activities in the business sector is determined in a similar way. The initial stock  $k_{T,1990}$  and the sequence of business

<sup>16</sup> Estimates of Corrado, Hulten, and Sichel (2005, 2006) are based on categories of expenditures, such as R&D and advertising, for which there are direct measures.



tangible investments imply the sequence of stocks  $\{k_{Tt}\}$  from (5). Equating the marginal products of tangible capital across the two business activities implies

$$k_{Tt}^1 = \left( \frac{y_{bt}}{y_{bt} + q_t x_{It}} \right) k_{Tt},$$

where  $k_{Tt}^2 = k_{Tt} - k_{Tt}^1$  is residually determined.

If we have a sequence for the price  $q_t$  of intangible investment, we could use the already computed sequence of outputs  $q_t x_{It}$  to determine a sequence for intangible investment  $x_{It}$ , and, with an initial condition for the stock  $k_{I,1990}$ , we could use (6) to determine the sequence of stocks.<sup>17</sup> But we do not have  $q_t$  and, therefore, we use the intertemporal condition

$$(10) \quad 1 = \beta \left( \frac{1 + \tau_{ct}}{1 + \tau_{c,t+1}} \right) \left( \frac{U_c(c_{t+1}, h_{t+1})}{U_c(c_t, h_t)} \right) R_{t,t+1},$$

where  $R_{t,t+1}$  is the after-tax return realized by the household investing in intangible capital,

$$\begin{aligned} R_{t,t+1} = & \{q_{t+1}(1 - \chi)(1 - \tau_{h,t+1})(1 - \delta_I) + (1 - \tau_{p,t+1})(1 - \tau_{d,t+1}) \\ & \times [q_{t+1}\chi(1 - \delta_I) + \phi(y_{b,t+1} + q_{t+1} x_{I,t+1})/k_{I,t+1}]\} / \{q_t[(1 - \chi)(1 - \tau_{h_t}) \\ & + \chi(1 - \tau_{p_t})(1 - \tau_{d_t})]\}, \end{aligned}$$

along with the capital accumulation equation and the already identified sequence for  $q_t x_{It}$ . This relation implicitly identifies  $q_{t+1}$  as a function of current variables and  $q_t$ . As a terminal condition, we assume that the price in the year following the end of our sample is equal to the price in the last year of our sample.

We should note that our construction of the paths for intangible capital and its price does not rely on any stock market data. An alternative approach used by Hall (2000) does. Specifically, Hall (2000) estimates the stock of intangible capital residually using the equilibrium relation

$$(11) \quad V_t = q_{Tt} K_{T,t+1} + q_{It} K_{I,t+1},$$

where  $q_{Tt}$  and  $q_{It}$  are prices of capital, along with data on the stock market capitalization of businesses  $V_t$  and reproducible costs of tangible capital  $K_{T,t+1}$ . A further difference between our estimates and Hall's is the modeling of the prices of capital. In our model, the prices of capital,  $q_{Tt}$  and  $q_{It}$ , are functions of tax rates and the relative price of intangible investment,  $q_t$ . In Hall's (2000) model, there are capital

<sup>17</sup> We use the steady-state stock to initialize intangible capital.

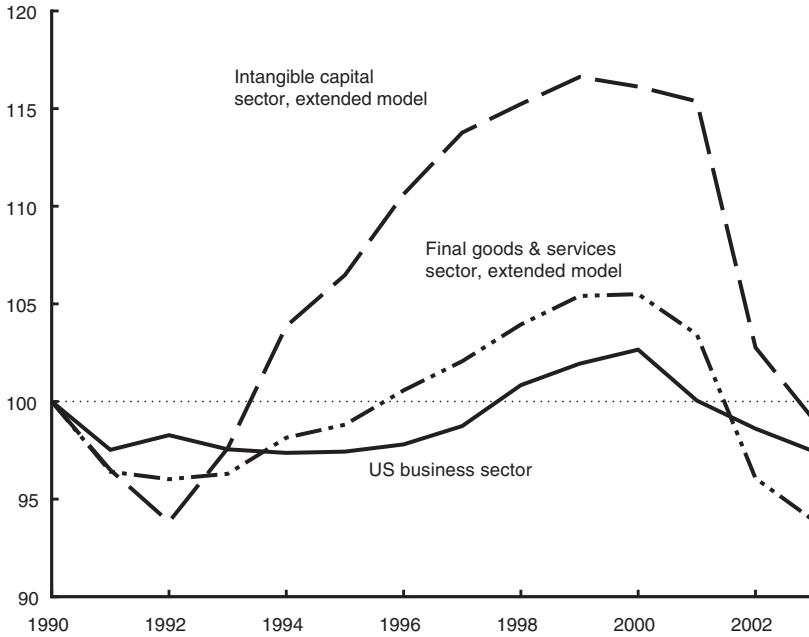


FIGURE 7. US AND EXTENDED MODEL REAL TOTAL FACTOR PRODUCTIVITY  
(Annual, series divided by  $1.02^t$ , 1990 = 100, 1990–2003)

adjustment costs and, as a consequence, the prices of capital are functions of net investment.<sup>18</sup> We chose not to follow Hall’s approach for estimating intangible capital because theory cannot account for excess volatility of stock prices: fluctuations in stock prices are many times greater than fluctuations in the price of capital. If intangible capital is found residually using (11), then the estimate is implausibly volatile.

The final step that we take in identifying the TFP parameters is to use the production functions (1) and (2) to determine  $\{A_t^1, A_t^2\}$ . The resulting sequences for these two TFPs are plotted in Figure 7 along with a more standard measure of business-sector TFP that abstracts from intangible capital: business value added divided by  $k_{Ti}^{0.33}(h_t - \bar{h}_m)^{0.67}$ , where  $k_{Ti}$  is tangible capital in the business sector and  $h_t - \bar{h}_m$  is total business hours. The latter measure is labeled “US business sector.” All series are real and divided by  $1.02^t$ .

The standard measure of business-sector TFP shows some acceleration beginning in 1996. The implied TFPs for the model with intangible investment show larger increases that begin earlier. For example, in the sector producing final goods and services, predicted growth in TFP between 1993 and 2000 is 0.7 percent greater per year than is found by constructing the standard measure. In the sector producing intangible capital, the implied growth in TFP between 1993 and 2000 is 2.7 percent greater per year than that found with the standard measure. All three measures show

<sup>18</sup> See, in particular, Hall’s (2000) equations (5) and (10), which provide two equations in two unknowns: the price of intangible capital and the intangible capital stock.



FIGURE 8. EXTENDED MODEL INTANGIBLE SHARE OF TOTAL OUTPUT  
(Annual, 1990–2003)

some decline after 2000, which could well have been due to regulatory and legal factors impinging negatively on efficiency.

The patterns of the TFP sequences are consistent with the micro and macro evidence we cite in Section II. For this reason, we view this theory as one that satisfies our exogenous input justification criterion. We next show that the theory satisfies our prediction criterion.

#### D. Model Predictions

Treating the TFP sequences as exogenous inputs, we compute the equilibrium path of all of the variables and compare them to US data. All of the parameters used in computing the equilibrium are described and motivated in Appendix B and summarized in Table B1 under the heading “Extended model, with intangible investment,” and in Table B2.<sup>19</sup>

In Figure 8, we display the implied expenditures on intangible investment  $q_t x_{It}$  as a share of total output, by which we mean GDP plus expenditures on intangible investment. This figure displays the bottom line of our study: the value of this investment is large and increased dramatically in the 1990s. That is precisely what we see in Figure 8. Our estimate of *net* intangible investment—expensed plus sweat—in

<sup>19</sup> Income shares listed in Table B1 are the same across activities. We do sensitivity checks on these shares and other parameters of the model to ensure that the main quantitative results are robust. See McGrattan and Prescott (2009).

the business sector is a little over 3 percent of total output—GDP plus intangible investment—in 1990 and rises to nearly 8 percent of total output before returning to the level of the early 1990s.<sup>20</sup> Decomposing the price and quantity, we find that the model predicts a 9 percent fall in the relative price  $q_t$  between 1990 and 2003 and a pattern for the quantity  $x_{it}$  similar to that for the expenditures  $q_t x_{it}$  shown in Figure 8.

We now assess the conformity of two sets of predictions. The first are predictions of variables used to identify the sequences of TFP in Figure 7. These series are hours and components of GDP. The second are predictions of variables that are not used to identify the sequences of TFP. These series are NIPA factor incomes and capital gains reported in the US Flow of Funds (1945–2005); the latter is especially important given the central role that capital gains play in our extended theory. We find that the model is in conformity with both sets of predictions.

*Internal Conformity.*—We start with a comparison to total hours and to components of GDP. Note that although we used equilibrium conditions to identify the TFP parameters, the predicted and actual series may differ because we used *only one of the two* intertemporal conditions when determining the TFP paths. Condition (10) relates the marginal rate of substitution in consumption between period  $t$  and  $t + 1$  to the after-tax return on investing in intangible capital. The second intertemporal condition, which was not used, relates the marginal rate of substitution in consumption to the after-tax return on investing in tangible capital. If the latter condition is not satisfied by the data, the predicted and actual paths will differ.

Figure 9 shows the results for per capita total hours worked. Unlike the comparable figure with the standard model's predictions (Figure 1), here, the predictions and the actual series track each other closely. The extended model predicts a fall in hours used to produce final goods and services during the 1990s. However, because hours spent building intangible capital rise significantly, the model predicts the large overall increase in hours worked  $h$ .

Similarly, the model and data paths for GDP are close. We plot these paths in Figure 10. The model's prediction for business value added is also close because, in the model, GDP is the sum of business value added  $y_b$  and nonbusiness value added  $\bar{y}_n$ , and the path for nonbusiness value added is preset to be the same as in the United States. In McGrattan and Prescott (2009), we display time series for consumption, tangible investment, and business labor productivity, which are all close in comparison to their US analogues.

*External Conformity.*—Now we consider a more demanding test of the theory: comparing model predictions to observations not used to determine the TFP paths. In particular, we compare predictions for business wage compensation as measured in the NIPA and for business capital gains as measured in the Flow of Funds accounts. We find that the model predicts these series remarkably well.

To compare the model's prediction for NIPA wage compensation in the business sector, we need to construct wages as a national accountant would. Such an

<sup>20</sup> The estimate of net intangible investment here exceeds earlier estimates in McGrattan and Prescott (2005b). In earlier work, we did not include sweat investment or noncorporate business activity.

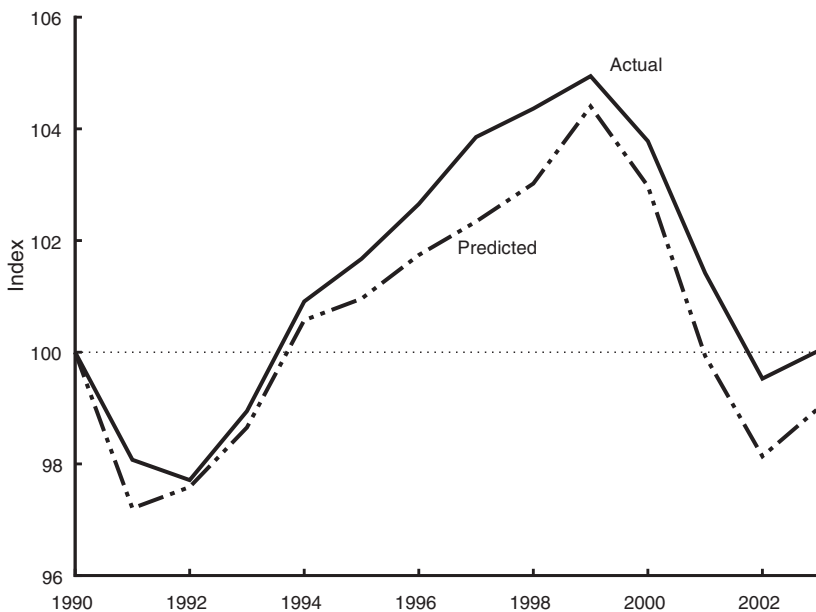


FIGURE 9. US AND EXTENDED MODEL PER CAPITA HOURS WORKED  
(Annual, 1990 = 100, 1990–2003)

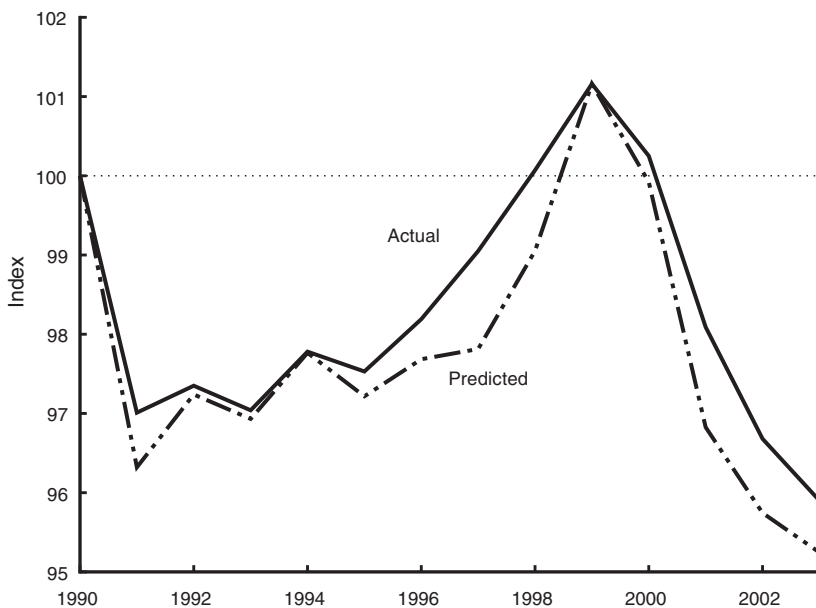


FIGURE 10. US AND EXTENDED MODEL PER CAPITA REAL GDP  
(Annual, series divided by 1.02', 1990 = 100, 1990–2003)

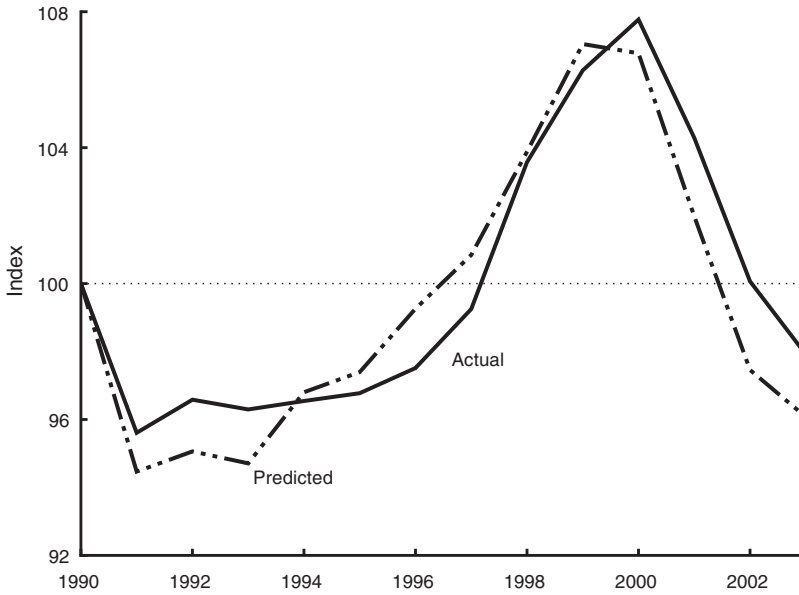


FIGURE 11. US AND EXTENDED MODEL REAL BUSINESS COMPENSATION LESS SWEAT  
(Annual, series divided by 1.02<sup>t</sup>, 1990 = 100, 1990–2003)

accountant, placed in the model economy, would report wage compensation in the business sector as  $w_t(h_t^1 + h_t^2) - (1 - \chi)q_t x_{It}$ , in effect, not including the value of sweat equity investment. In Figure 11, we plot this predicted series along with the actual US series. Both are real series, detrended by 2 percent annually and set equal to 100 in 1990. The two are close. Relative to the 1990 trend level, both the model prediction and the actual wages are up nearly 8 percent in 2000. We note that our choice of  $\chi = 0.5$  is relevant for this prediction. The value of  $\chi$  determines the level of taxation on expensed versus sweat equity, which affects the equilibrium measured compensation. Higher values of  $\chi$  increase the predicted value of compensation. We selected  $\chi = 1/2$  given that we do not have independent evidence of the financing of expensed and sweat equity. In McGrattan and Prescott (2009), we show that our results are not sensitive to the choice of  $\chi$  unless  $\chi$  is far from 1/2. For example, if we assume that all intangible investment is financed by shareholders ( $\chi = 1$ ), then the model overpredicts wages by about 3 percentage points in 2000.

Next, we compare the model's predictions for estimates of the increase in capital gains from expensed and sweat equity to US household holding gains reported in the Flow of Funds accounts. Those gains are the change in the value of assets outstanding (taken from table L.100) less the net purchases during the period (taken from table F.100). If Flow of Funds accountants recorded holding gains for our model households, they would compute differences in the total value of businesses (for which the household is the residual claimant). The value of all businesses in  $t$ ,  $V_t$ , is composed of two parts:

$$(12) \quad V_t = (1 - \tau_{dt})K_{T,t+1} + [\chi(1 - \tau_{dt})(1 - \tau_{pt}) + (1 - \chi)(1 - \tau_{ht})]q_t K_{I,t+1},$$

where capital letters denote aggregates. On the right side of (12), the first term is the value of tangible capital and the second is the value of intangible capital. Notice that the price of intangible capital depends on  $\chi$ , since income to capital and income to labor are taxed differently.

The change in the value  $V_t$  of businesses does not exactly reflect the additional income in the model economy. The additional income is  $q_t X_{It}$  (in units of the final goods and services). However, during periods with large investments of intangible capital, the increase in holding gains, as defined in the Flow of Funds accounts, is a good approximation to the increase in intangible investment.

To make our estimates of the gains comparable with the US Flow of Funds measure, we must adjust our model estimate to account for the fact that some capital of US firms is abroad and some domestic capital is owned by US affiliates of foreign corporations. To do this, we assume that the ratio of after-tax foreign corporate profits to after-tax domestic corporate profits is equal to the ratio of foreign to domestic holding gains. The corporate profits data are taken from NIPA and exclude income from capital gains. The foreign corporate profits are net of profits made by US affiliates of foreign corporations. With this adjustment, our estimate of net foreign holding gains is 28 percent of domestic gains on average for the period 1990–2003.

A significant break in US real holding gains (relative to GDP) occurred in 1995. Before that year, the series averages around 6 percent of GDP. In 1995 and thereafter, the average is 12 percent. A difference of 6 percent of GDP is economically large. To determine whether the difference is statistically significant, we ran the following statistical test. We ran the regression

$$g_i = \alpha + \beta d_i,$$

where  $g_i$  is the real gain in decade  $i$  and  $d_i$  is a dummy variable, which is 1 for the 1995–2004 decade and 0 for the four decades preceding it (which is when data are available). Our estimate for  $\beta$  is 6.2 percent with a standard error of 2.9 percent, indicating that the change in the mean is statistically significant.

In Figure 12, we plot average real holding gains relative to GDP for the United States along with the extended model's prediction for them.<sup>21</sup> Both curves rise significantly in the late 1990s. The rise is coincident with the dramatic rise in hours.

These results lead us to the conclusion that the model satisfies our prediction criterion.

### E. *Is Success Guaranteed?*

A crucial element of the theory is intangible investment, which is directly measurable only in part. We treat the total as unmeasurable and, therefore, unobserved. Does this imply that intangible investment is simply making up for whatever is missing in the standard theory? In this section (and more fully in McGrattan and Prescott 2009), we demonstrate that intangible capital per se does not resolve the

<sup>21</sup> See McGrattan and Prescott (2009) for the nonaveraged series.

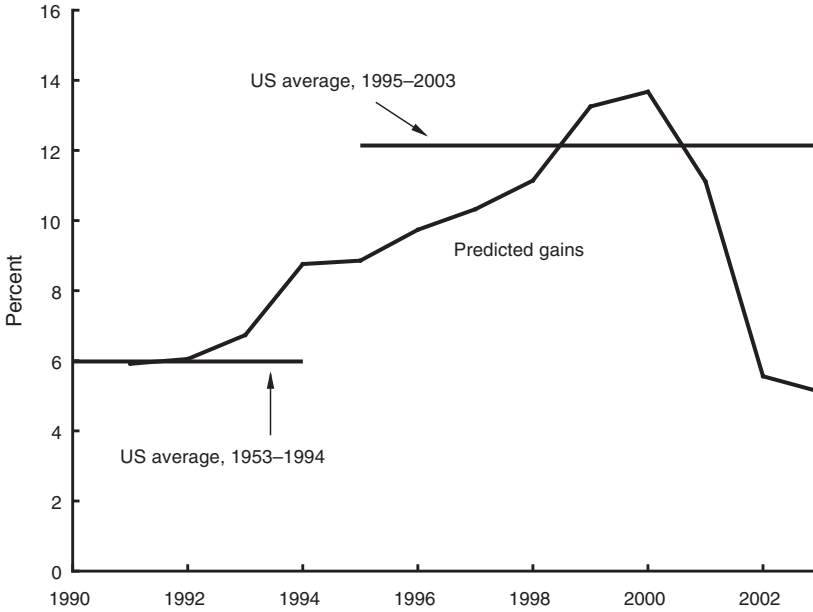


FIGURE 12. US AND EXTENDED MODEL REAL HOLDING GAINS  
(Annual, percent of real GDP, excluding real estate, 1990–2003)

puzzling 1990s. If we extend the basic neoclassical model to include intangible capital but assume that technological change is *neutral*, then the theory does not satisfy the two criteria we require to successfully resolve the puzzling 1990s boom.

In the alternative version of the model, we introduce a sequence of labor wedges  $\{L_{wt}\}$  that are chosen so that the household's intratemporal first-order condition is satisfied,

$$\frac{\psi(1 + \tau_{ct})c_t}{1 - h_t} = L_{wt}(1 - \tau_{ht})w_t,$$

and assume that the TFPs in (1) and (2) vary proportionally. The wedges are proxies for labor distortions other than government taxes on labor. If the income shares are common in the two activities, as assumed above, the relative price of intangible investment  $q$  is constant. We normalize it to one. As above, there are two “free” parameter sequences. In this case, they are  $\{A_t^1, L_{wt}\}$ .

We find the implications of the alternative model are grossly at odds with what is reasonable behavior for labor distortions and intangible investments. The resulting sequences of  $\{L_{wt}\}$  and  $\{x_{It}\}$  oscillate wildly. For example, the series for the labor wedge oscillates between 0.8 and 1.4 and displays little persistence. Given what we know about labor markets, this pattern is unreasonable. Furthermore, an hours boom is generated only if TFP and capital tax rates are also oscillatory and offsetting, which is unreasonable in the case of TFP and counterfactual in the case of capital tax rates.

In summary, it is not the inclusion of intangible capital per se that resolves the puzzling US boom in the 1990s. We find that including *both* intangible capital and



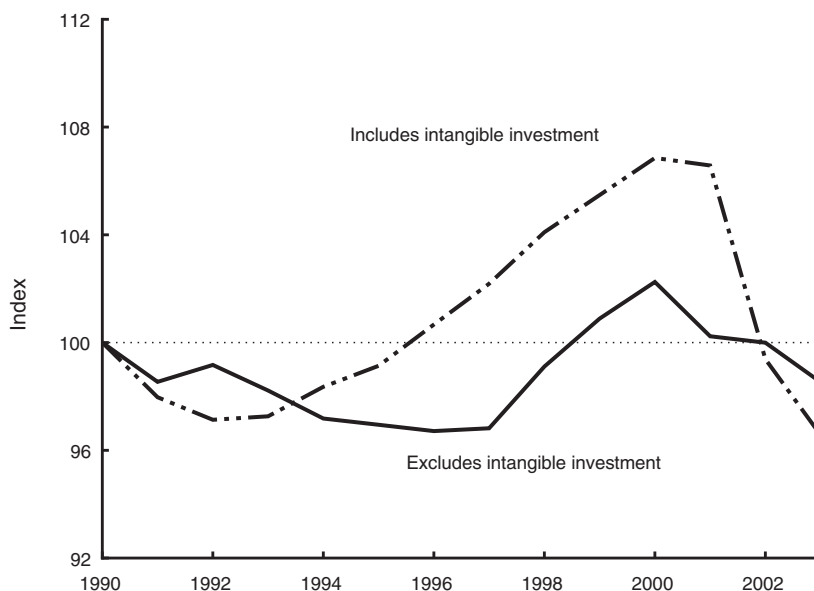


FIGURE 13. EXTENDED MODEL REAL BUSINESS PRODUCTIVITY  
(Annual, series divided by  $1.02^t$ , 1990 = 100, 1990–2003)

non-neutral technological change resolves the puzzle. We turn now to using our theory as a tool for uncovering what actually did happen in the 1990s.

#### IV. Reevaluation of the US Economy in the 1990s

What does all of this mean for US labor productivity and investment? If some output is unmeasured relative to inputs, then GDP and productivity estimates are biased downward. If the missing output is expensed or sweat investment, then the investment estimates are also biased downward. Our extended model's predictions for variables with and without intangible investment demonstrate how distorted standard data and models are for assessing the 1990s.

In Figure 13, we compare two predictions for business labor productivity, both computed from the extended model. One is the model's prediction for business value added—without intangible investment included—divided by total business hours. This is what a national accountant would construct. The other includes intangible investment as part of business value added. Both series are detrended by 2 percent annually and set equal to 100 in 1990. Notice how different the predictions are. Measured labor productivity, which is what national accountants would record, shows a significant fall relative to trend up to 1997 and then a sharp increase through 2000. But true productivity, including intangible investment, fell only until 1993 and then, starting in 1994, grew very quickly. Over the period 1993–2000, the difference in growth rates for these two series is 0.8 percent per year.

Oliner, Sichel, and Stiroh (2007) include a measure of intangible capital in their analysis of labor productivity growth in the nonfarm business sector over the

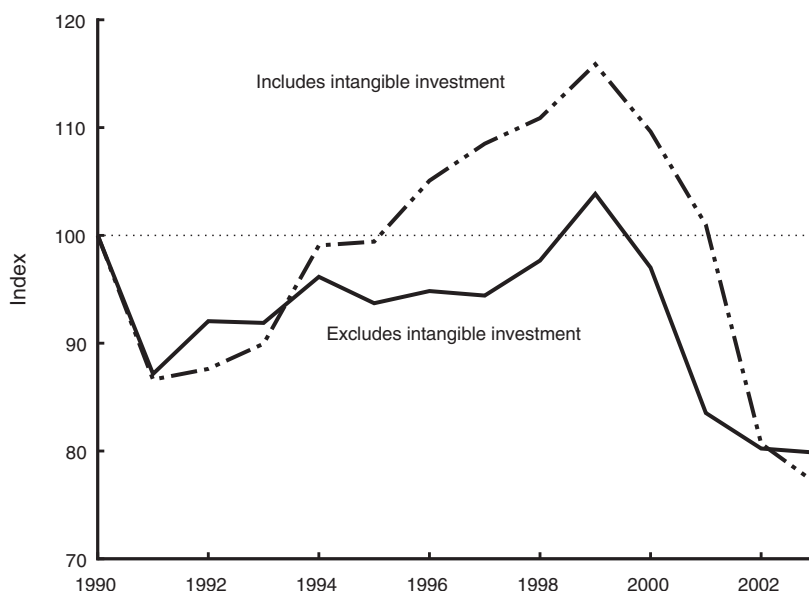


FIGURE 14. EXTENDED MODEL REAL PER CAPITA INVESTMENT  
(Annual, series divided by 1.02<sup>t</sup>, 1990 = 100, 1990–2003)

period 1973 to 2006. In the 1995–2000 subperiod, they compare the growth rate of labor productivity when they account for intangibles to the published estimate of the BLS which excludes intangibles. Their estimate is 2.95 percent per year, while the BLS estimates 2.51 percent per year. If we restrict attention to the 1995–2000 period and compare our model predictions with and without intangibles, then we predict a labor productivity growth annual rate for the business sector of 3.51 percent when we include intangible investment and 3.07 percent when we exclude intangible investment. The levels of the two sets of estimates are different because the coverage and sources of the underlying data are different.<sup>22</sup> However, the impact of adding intangibles is estimated to be the same: 0.44 percent per year over the period 1995–2000.

In Figure 14, we compare the model's two measures of total investment: tangible investment and tangible plus intangible investment. Again, both are detrended by 2 percent annually and normalized to 100 in 1990. And again, the predictions—with and without intangible investment—are very different. Between 1991 and 1999, tangible investment rose almost 20 percent. Total investment, however, rose more than 30 percent.

In summary, our results show that standard accounting measures and predictions of models without intangible investment do not accurately reflect what was going on in the US economy during the 1990s.

<sup>22</sup> Oliner, Sichel, and Stiroh (2007) use the nonfarm business sector and the hours data based on the establishment survey. We use the business sector as defined in Appendix A and hours data based on the CPS.

## V. Conclusion

We find that non-neutral technological change in the production of intangible investment goods was what gave rise to the puzzling behavior of the US economy in the 1990s. This change resulted in a boom in intangible investment, which is not included in the national accounts measure of output. Once this feature of reality is introduced into the basic neoclassical growth model to obtain what we call the extended growth model, we see that the US economy in the 1990s is in close conformity with theory for both the income and product sides of the national accounts and for the jump in average accrued capital gains. Furthermore, microeconomic evidence is broadly consistent with the view that investments in intangible capital increased relative to GDP in the 1990s.

The important implication of our analysis is that our extended growth model with intangible investments should be used in aggregate economic analyses. Indeed, we see it as a significant improvement on the basic neoclassical growth model.

A puzzle that is as yet unsolved is why the technology bust of the early 2000s was so severe, especially for investments and asset values of IT companies. The analysis in the paper assumes that agents have perfect information about the productivity and output of intangible investments, which implies that the source of the technology bust and large decline in hours of work is a fall in that activity's productivity parameter. An extension of our analysis would allow for imperfect information about the current output of intangible capital, which results in measurement errors in that activity's productivity parameter. It is possible that measurement errors led to an overestimate of total factor productivity in the late 1990s and, as a consequence, high levels of investment and hours. The sharp decline in our measure of TFP at the beginning of the 2000s could be an artifact of these errors being recognized.

## APPENDIX A: DATA

The three main sources for our data are the BEA, which publishes the national accounts and fixed asset tables; the Federal Reserve Board, which publishes the Flow of Funds tables; and the BLS, which publishes data on hours and population. In this Appendix, we provide details on the specific data we use and the necessary revisions we make so that the data are consistent with growth theory.

### I. National Accounts and Fixed Assets

*Overview and Sources.*—Tables A1–A3 contain a summary of the revised national accounts along with averages for the period 1990–2003. The table numbers and sources of the raw data are listed in parentheses. The sources are tables from the BEA's NIPA and fixed asset (FA) tables, and the Federal Reserve's Flow of Funds (FOF) accounts. For example, NIPA 7.5 is table 7.5 from the BEA NIPA tables. The averages shown in the table for the period we study are the means of nominal shares of nominal GDP and are included to help with magnitudes in each category. When

TABLE A1—REVISED NATIONAL ACCOUNTS, AVERAGES RELATIVE TO GDP, 1990–2003  
(domestic business value added)

1	Domestic business value added	0.748
2	Consumption of fixed capital	0.082
3	Corporate business (NIPA 7.5)	0.067
4	Sole proprietorships and partnerships (NIPA 7.5)	0.013
5	Other private business (NIPA 7.5)	0.003
6	Labor income	0.494
7	Compensation of employees	0.421
8	Corporate business (NIPA 1.13)	0.382
9	Sole proprietorships and partnerships (NIPA 1.13)	0.036
10	Other private business (NIPA 1.13)	0.002
11	70 percent proprietors' income with IVA and CCadj (NIPA 1.13)	0.049
12	Sweat investment (authors' calculations)	0.024
13	Capital income	0.172
14	Corporate profits with IVA and CCadj (NIPA 1.13)	0.073
15	30 percent proprietors' income with IVA and CCadj (NIPA 1.13)	0.021
16	Rental income of persons with CCadj (NIPA 1.13)	0.006
17	Net interest and miscellaneous payments	0.022
18	Corporate business (NIPA 1.13)	0.014
19	Sole proprietorships and partnerships (NIPA 1.13)	0.012
20	Other private business (NIPA 1.13)	0.005
21	Less: Intermediate financial services <sup>a</sup> (NIPA 2.5.5)	0.009
22	Taxes on production and imports <sup>b</sup>	0.026
23	Corporate business (NIPA 1.13)	0.056
24	Sole proprietorships and partnerships (NIPA 1.13)	0.008
25	Other private business (NIPA 1.13)	0.002
26	Less: Sales tax (NIPA 3.5)	0.040
27	Expensed investment (authors' calculations)	0.024

Note: IVA, inventory valuation adjustment; CCadj, capital consumption adjustment.

<sup>a</sup>Expense is for handling life insurance and pension plans.

<sup>b</sup>This category includes business transfers and excludes subsidies.

we compare model predictions with data, we work with real measures and deflate all nominal US time series by the NIPA GDP implicit price deflator.

Tables A1 and A2 are the income side of our revised accounts. In Table A1, we display the components of our measure of domestic business value added. This measure is close to, but not exactly the same as, the sum of the value added of corporate business, sole proprietorships and partnerships, and other private business as defined in the NIPA tables. In Table A2, we display the components of our measure of domestic nonbusiness value added. This measure is close to, but not exactly the same as, the sum of value added of the household business sector, nonprofits, general government, and government enterprises. Table A3 provides details of the product side of the accounts along with totals for the income side (for comparison). We have categorized tangible investment into business and nonbusiness as in the case of incomes. That is, investments of corporations and noncorporate business are included with business investment, and investments of household business, nonprofits, and government are included with nonbusiness investment.

Data on capital stocks are used to impute some services of capital when we revise the accounts. They are also used to set certain model parameters and to initialize stocks when computing model equilibria. We use BEA reproducible stocks (FA

TABLE A2—REVISED NATIONAL ACCOUNTS, AVERAGES RELATIVE TO GDP, 1990–2003  
(domestic nonbusiness value added)

1	Domestic nonbusiness value added	0.337
2	Consumption of fixed capital	0.099
3	Households	0.074
4	Excluding consumer durables (NIPA 7.5)	0.012
5	Consumer durable depreciation (FOF F10)	0.062
6	Nonprofits (NIPA 7.5)	0.004
7	General government (NIPA 7.5)	0.018
8	Government enterprises (NIPA 7.5)	0.003
9	Labor income	0.154
10	Compensation of employees	0.154
11	Households (NIPA 1.13)	0.001
12	Nonprofits (NIPA 1.13)	0.042
13	General government (NIPA 1.13)	0.099
14	Government enterprises (NIPA 1.13)	0.012
15	Capital income	0.084
16	Current surplus of government enterprises (NIPA 1.13)	0.001
17	Rental income of persons with CCadj (NIPA 1.13)	0.008
18	Net interest and miscellaneous payments	0.033
19	Households (NIPA 1.13)	0.031
20	Nonprofits (NIPA 1.13)	0.002
21	Taxes on production and imports <sup>b</sup>	0.004
22	Households (NIPA 1.13)	0.011
23	Nonprofits (NIPA 1.13)	0.001
24	Less: Sales tax (NIPA 3.5)	0.007
25	Imputed additional capital services <sup>c</sup>	0.038
26	Household, consumer durables	0.013
27	Government capital	0.025

Note: IVA, inventory valuation adjustment; CCadj, capital consumption adjustment.

<sup>a</sup>Expense is for handling life insurance and pension plans.

<sup>b</sup>This category includes business transfers and excludes subsidies.

<sup>c</sup>Imputed additional capital services are equal to 4.1 percent times the current-cost net stock of government fixed assets and consumer durables goods (FA 1.1).

table 1.1 for totals and FA table 6.1 by owner). To that we add land values based on Federal Reserve market values of real estate from balance sheets of households (FOF B100), nonfarm nonfinancial corporations (FOF B102), and nonfarm noncorporate (FOF B103). For farmland, we follow Gary D. Hansen and Prescott (2002) and assume it is roughly 0.08 times GDP.

*Revisions.*—We now describe how we revise the national accounts to make them consistent with our model. Three adjustments are necessary in the models with or without intangible investment. A fourth adjustment is necessary when we include intangible investment.

**Consumption Taxes:** Unlike the NIPA, our model output does not include consumption taxes as part of consumption and as part of value added. We thus subtract sales and excise taxes from the NIPA data on taxes on production and imports (Table A1, line 26; Table A2, line 24) and from personal consumption expenditures (Table A3, line 10) since these taxes primarily affect consumption expenditures. As a result of this adjustment, we use producer prices rather than a mixture of producer and consumer prices.

TABLE A3—REVISED NATIONAL ACCOUNTS, AVERAGES RELATIVE TO GDP, 1990–2003  
(domestic value added and product)

1	Total adjusted domestic income	1.091
2	Domestic business value added	0.748
3	Domestic nonbusiness value added	0.337
4	Statistical discrepancy	0.006
5	Total adjusted domestic product	1.091
6	Private consumption	0.618
7	Personal consumption expenditures (NIPA 1.1.5)	0.678
8	Less: Consumer durables (NIPA 1.1.5)	0.083
9	Less: Intermediate financial services <sup>a</sup> (NIPA 2.5.5)	0.009
10	Less: Sales tax, nondurables and services (NIPA 3.5)	0.042
11	Consumer durable depreciation (FOF F10)	0.062
12	Imputed additional capital services <sup>c</sup>	0.013
13	Public consumption (NIPA 3.1)	0.179
14	Government consumption expenditures (NIPA 3.1)	0.154
15	Imputed additional capital services <sup>c</sup>	0.025
16	Business tangible investment <sup>d</sup>	0.112
17	Corporate gross private domestic investment (FOF F6)	0.092
18	Noncorporate gross private domestic investment (FOF F6)	0.020
19	Nonbusiness tangible investment	0.134
20	Household	0.114
21	Excluding consumer durables (FOF F6)	0.036
22	Consumer durables (NIPA 1.1.5)	0.083
23	Less: Sales tax, durables (NIPA 3.5)	0.005
24	Nonprofits (FOF F6)	0.007
25	Government investment (NIPA 3.1)	0.033
26	Net exports of goods and services (NIPA 1.1.5)	−0.021
27	Business intangible investment (authors' calculations)	0.048

Note: IVA, inventory valuation adjustment; CCadj, capital consumption adjustment.

<sup>a</sup>Expense is for handling life insurance and pension plans.

<sup>b</sup>This category includes business transfers and excludes subsidies.

<sup>c</sup>Imputed additional capital services are equal to 4.1 percent times the current-cost net stock of government fixed assets and consumer durables goods (FA 1.1).

<sup>d</sup>10 percent of farm business is in corporate, with the remainder in noncorporate.

**Financial Services:** We treat some of the NIPA's financial services as intermediate rather than as final and, therefore, need to subtract them from GDP and from consumption services. Specifically, we subtract personal business expenses for handling life insurance and pension plans from net interest (Table A1, line 21) and from personal consumption expenditures (Table A3, line 9).

**Fixed Asset Expenditures:** We treat expenditures on all fixed assets as investment. Thus, spending on consumer durables is treated as an investment rather than as a consumption expenditure and moved from private consumption (Table A3, line 8) to nonbusiness tangible investment (Table A3, line 22). We introduce a consumer durables services sector in much the same way as the NIPA introduces owner-occupied housing services. Households rent the consumer durables to themselves. Specifically, we add depreciation of consumer durables to consumption of fixed capital of households (Table A2, line 5) and to private consumption (Table A3, line 11). We add imputed additional capital services for consumer durables to capital income

(Table A2, line 26) and to private consumption (Table A3, line 12). We assume a rate of return equal to 4.1 percent, which is an estimate of the return on other types of capital. A related adjustment is made for government capital. Specifically, we add imputed additional capital services for government capital to capital income (Table A2, line 27) and to public consumption (Table A3, line 15).

**Intangible Investment:** We introduce intangible investment in the growth model in Section III. Our output measure includes intangible investment. Thus, total product in the model is the sum of intangible investment and GDP (which we define to be NIPA GDP *after* adjustments are made for consumption taxes, intermediate financial services, and consumer durables).<sup>23</sup> On the income side of our extended model accounts, we add capital gains  $q_t x_{it}$ . Fraction  $\chi$  of these gains is “expensed investment,” which is allocated to capital income (Table A1, line 27). Fraction  $1 - \chi$  of these gains is “sweat investment,” which is allocated to labor income (Table A1, line 12). On the product side, we add “business intangible investment” (Table A3, line 27). In Section III, we describe our calculations.

After the adjustments above are made to the nominal US series, we detrend them by dividing by three factors: the NIPA GDP implicit price deflator; the population series (defined below); and the factor  $1.02^t$  to account for growth in technology.

*Tax Rates on Consumption and Labor.*—We use data from the US national accounts to construct estimates for the tax rates on consumption and labor.

The tax rate on consumption is found by taking the ratio of sales taxes in NIPA to consumption expenditures in NIPA (which include sales taxes). In our measure of sales taxes, we include federal excise taxes and customs, state and local sales taxes, and other non-property licenses and fees. Our measure of NIPA consumption expenditures includes adjustments for consumer durables. Denoting sales tax by  $\tau_c c$  and NIPA consumption expenditures by  $c + \tau_c c$ , the ratio yields  $\tau_c / (1 + \tau_c)$ . It is easy to determine  $\tau_c$  from this ratio.

For the marginal tax rate on labor, we apply essentially the same methodology as in Prescott (2004). Specifically, we take the effective labor tax to be the sum of a marginal income tax rate and a marginal tax rate for social security. The income tax rate is computed as follows. Take personal current taxes in NIPA (which are direct taxes paid by households) and divide them by GDP plus net gain from sale of assets less depreciation and taxes on production and imports. We include gains from asset sales because personal current taxes include taxes on these gains. Prescott (2004) multiplies the income tax rate by 1.6 in order to get estimates of the *marginal* rate comparable to Daniel Feenberg and Elisabeth Coutts (1993). The marginal tax rate on social security is computed as follows. Take contributions for government social insurance in NIPA and divide them by labor income. For labor income, we sum compensation and 70 percent of proprietors’ income.

<sup>23</sup> In 1999, the BEA recognized expenditures for computer software as investment. In McGrattan and Prescott (2009), we show that excluding software from GDP and including it with intangible investment has little impact on our quantitative results.

## II. Hours and Population

The primary source of our hours and population data is the US Department of Labor, BLS, *Employment and Earnings*. They are based on the CPS. We briefly describe these data here. Full details are given in Prescott, Alexander Ueberfeldt, and Simona Cociuba (2005).

The population covered by our series is the total noninstitutional population, ages 16–64, for the United States. Military hours are estimated and added to civilian hours from the CPS.

For versions of the growth model with business and nonbusiness sectors, we also categorize CPS hours as business and nonbusiness. Using the March supplement (through [www.ipums.org](http://www.ipums.org)), we construct business hours as the sum of hours for the self-employed—both incorporated and unincorporated—and hours for private wage and salary workers less hours for employees in nonprofits. Because private wage and salary workers include employees at nonprofits, we use BEA data on compensation in nonprofits, and assuming an average wage rate equal to the economy-wide average, we can infer hours for nonprofits. Hours in the nonbusiness sector are found by subtracting business hours from the total. We use the hours from the March supplement sample to compute the fractions of hours in business and nonbusiness. For our final series, we multiply these fractions by total hours in the monthly CPS sample.

### APPENDIX B: PARAMETERS

In this Appendix, we report and motivate the parameters and exogenous technology and tax series used as inputs for the three models described in the main text. The parameter values are summarized in Table B1. The exogenous technology and tax rate series are summarized in Table B2. In a separate technical Appendix (McGrattan and Prescott 2009), we also demonstrate, by doing sensitivity analysis, that our main results are robust.

For interest and growth rates, we use estimates based on US trends. In particular, we set the interest rate at 4.1 percent (as in McGrattan and Prescott 2005b) and the annual growth in population  $\eta$  at 1 percent. We also assume that per capita GDP and its components grow at 2 percent annually ( $\gamma = 0.02$ ). These choices imply that  $\beta = 0.98$ . These parameters are used in all versions of our model and ensure that the marginal rate of substitution is the same across experiments.

In all three models, we use constant tax rates on capital. In both versions of the growth model in which we distinguish business and nonbusiness activity, we set the profits tax rate to  $\tau_p = 0.35$  because most of the taxes on profits are corporate income taxes. In both cases, we set the distribution tax  $\tau_d = 0.15$ , which is slightly less than our estimate in earlier work (McGrattan and Prescott 2005b) for corporate distributions; this is appropriate because noncorporate incomes are not taxed twice. In the one-sector version of the growth model, we set the rates lower because most of the capital is not in the business sector and not affected by  $\tau_p$  or  $\tau_d$ . In fact, we set these rates equal to 0.42 times the rates used in the business sector, since business tangible capital is 42 percent of total tangible capital. For property tax rates, we use NIPA property tax revenues (in “taxes on imports and production”) to infer values



TABLE B1—MODEL PARAMETERS

Parameter	Expression	Value
Common parameters		
Growth in population	$\eta$	0.01
Growth in technology	$\gamma$	0.02
Discount factor	$\beta$	0.98
One-sector model, no intangible investment		
Utility parameter	$\psi$	1.48
Depreciation rate	$\delta$	0.031
Capital share	$\theta$	0.34
Tax rate on property	$\tau_k$	0.0073
Tax rate on profits	$\tau_p$	0.15
Tax rate on distributions	$\tau_d$	0.064
Two-sector model, no intangible investment		
Utility parameter	$\psi$	1.38
Depreciation rate, business	$\delta$	0.033
Capital share, business	$\theta$	0.28
Tax rate on business property	$\tau_k$	0.014
Tax rate on business profits	$\tau_p$	0.35
Tax rate on business distributions	$\tau_d$	0.15
Extended model, with intangible investment		
Utility parameter	$\psi$	1.32
Tangible depreciation rate, business	$\delta_T$	0.033
Intangible depreciation rate, business	$\delta_I$	0
Tangible capital share, business	$\theta$	0.26
Intangible capital share, business	$\phi$	0.076
Tax rate on business property	$\tau_k$	0.014
Tax rate on business profits	$\tau_p$	0.35
Tax rate on business distributions	$\tau_d$	0.15
Fraction of intangible expensed from profits	$\chi$	0.5

TABLE B2—TIME SERIES FOR TAX RATES AND TECHNOLOGY

Year $t$	Technology parameters					
	Tax rates		One-sector model	Two-sector model	Extended model	
	$\tau_{ct}$	$\tau_{ht}$			$A_t^1$	$A_t^2$
1990	6.6	31.1	1.49	1.75	1.66	1.53
1991	6.8	30.7	1.47	1.71	1.60	1.48
1992	6.8	30.3	1.48	1.73	1.60	1.44
1993	6.8	30.3	1.46	1.71	1.60	1.49
1994	7.0	30.7	1.45	1.71	1.63	1.59
1995	6.9	31.2	1.44	1.71	1.64	1.63
1996	6.7	31.9	1.44	1.71	1.67	1.69
1997	6.7	32.5	1.44	1.73	1.69	1.74
1998	6.7	33.3	1.45	1.76	1.73	1.77
1999	6.6	33.4	1.46	1.78	1.76	1.79
2000	6.5	34.3	1.46	1.80	1.76	1.79
2001	6.3	34.7	1.45	1.76	1.73	1.78
2002	6.2	30.8	1.45	1.73	1.60	1.59
2003	6.2	28.9	1.43	1.71	1.56	1.53

for  $\tau_k$  in the one-sector and two-sector versions of the model. Details of our labor and consumption tax rates are provided in Appendix A.

For the remaining parameters, we use 1990 levels of US data to obtain estimates. (See Appendix A for a full description of the data.) Specifically, we use the

consumption share of GDP, the tangible investment share of GDP, the total and business tangible capital stocks (including land) as shares of GDP, and hours as a fraction of discretionary time. We use 52 weeks times 100 hours per week as an estimate of discretionary time.

The ratio of tangible investment to the stock is used to infer the rate of depreciation of tangible capital in total or, if there are two sectors, in business. In Table B1, we report estimates of depreciation for each of the three models. These rates are slightly lower than is typical in the literature because we include land and inventories in our estimates of the capital stock. If we do not include them, the estimates for annual depreciation are on the order of 5 or 6 percent. There is no way to determine  $\delta_r$ . For this rate, we chose 0 and experimented with other values to make sure our main results did not change.

Given an interest rate, tax rates on capital, and the ratio of tangible capital to output, we can infer the share of tangible capital in producing final goods and services in the models without intangible investment.<sup>24</sup> For the one-sector version of the model, we find a capital share of 0.34. For the two-sector version, the capital share in business is 0.28. This is lower because the nonbusiness sector is more capital intensive.

In the extended model, we need more information because we do not know the split of tangible capital in final goods production and intangible capital production. Here, we use *reported* NIPA compensation for 1990, which, in theory, is equal to total compensation less sweat investment. Using the fact that input elasticities for producing both final goods and intangible capital are assumed to be the same along with information on NIPA compensation allows us to determine all capital shares. (See McGrattan and Prescott 2009 for details.) In Table B1, we report that the shares for tangible capital in production are 0.26, slightly lower than that for the two-sector model with no intangible investment. The intangible shares are 0.076.

The household's intratemporal condition, along with US observables and estimates of the capital shares and tax rates, implies a value for the utility parameter  $\psi$ . In Table B1, we report the values for each of the models. They are in the range of estimates used in the business cycle literature.

The final parameter to be set is  $\chi$ , the fraction of intangible investment that is financed by capital owners. This parameter is used only in our extended growth model with intangible investment. As noted earlier, the only real ramification of this choice is for tax payments. But the evidence in Figures 2 and 3 indicates that some investment is being done by both shareholders and workers. We chose  $\chi = 0.5$  and then experimented with other values. The main effect of varying  $\chi$  is a change in the effective tax rates on labor and capital.

<sup>24</sup> An alternative procedure uses information on factor incomes to infer cost shares. However, this procedure is invalid when there are intangible investments because the NIPA accounts do not report the total compensation or profits.

## APPENDIX C: AGGREGATION IN THE EXTENDED MODEL

In this Appendix, we develop the aggregation theory underlying the technology of our extended growth model with intangible investment (see Section III).

A business is characterized by the stock of its (unmeasured) intangible capital,  $K_I$ . This capital can be used for two activities. One activity produces the composite output of the business  $Y_b$ , and the other produces intangible investment goods  $X_I$ .

Inputs of (measured) tangible capital  $K_T^i$  and hours  $H^i$  along with  $K_I$  produce an intermediate good  $Z^i$  via a standard constant returns to scale neoclassical production function  $f^i$  for  $i \in \{1, 2\}$ . In particular, the production functions are

$$Z^i = (K_T^i)^\theta K_I^\phi (H^i)^{1-\theta-\phi}, i \in \{1, 2\}.$$

The quantity of  $Y_b$  produced is  $g^1(Z^1)$ , and the quantity of  $X_I$  produced is  $g^2(Z^2)$ . The functions  $g^i$  are increasing, initially strictly convex, then strictly concave, and they satisfy  $g^i(0) = 0$ . The slope of the maximal tangent ray from the origin is  $A^i$ . The point of tangency is  $\hat{Z}^i$ . The margin of adjustment is the number of units operated, which is variable. The capital stock  $K_I$  can be split over businesses through mergers, acquisitions, and spin-offs. All production units that are operated will have the same  $K_I$ . This  $K_I$  will depend upon the relative prices of the three inputs. Production units of type  $i$  will be operated at level  $\hat{Z}^i$  and produce  $g^i(\hat{Z}^i)$ .

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